ProofWatch++: Watchlist Guidance for E prover (and some Machine Learning)

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ITP'18 and BUMERANG'18, 24th September 2018

Learning From Mizar Proofs

We work with

- Mizar Mathematical Library

```
theorem :: WAYBEL_1:85
  for H being non empty lower-bounded RelStr st H is Heyting holds
  for a, b being Element of H holds 'not' (a "/\" b) >= ('not' a) "\/" ('not' b)
```

```
theorem Th36: :: YELLOW_5:36
  for L being non empty Boolean RelStr
  for a, b being Element of L holds
  ( 'not' (a "\/" b) = ('not' a) "/\" ('not' b) & 'not' (a "/\" b) = ('not' a) "\/" ('not' b) )
```

Learning From Mizar Proofs: WAYBEL_1 85

```
theorem :: WAYBEL_1:85
  for H being non empty lower-bounded RelStr st H is Heyting holds
  for a, b being Element of H holds 'not' (a "/\" b) >= ('not' a) "\/" ('not' b)
```

```
  let H be non empty lower-bounded RelStr ; :: thesis:
  assume A1: H is Heyting ; :: thesis:
  then A2: Bottom H <= Bottom H by ORDERS_2:1;
  let a, b be Element of H; :: thesis:
  A3: 'not' a <= 'not' a by A1, ORDERS_2:1;
  A4: 'not' b <= 'not' b by A1, ORDERS_2:1;
  (a "/\" b) "/\" (('not' a) "\/" ('not' b)) = ((a "/\" b) "/\" ('not' a)) "\/" ((a
  "/\" b) "/\" ('not' b)) by A1, Def3
  .= ((b "/\" a) "/\" ('not' a)) "\/" ((a "/\" b) "/\" ('not' b)) by A1, LATTICE3:15
  .= (b "/\" (a "/\" ('not' a))) "\/" ((a "/\" b) "/\" ('not' b)) by A1, LATTICE3:16
  .= (b "/\" (a "/\" ('not' a))) "\/" (a "/\" (b "/\" ('not' b))) by A1, LATTICE3:16
  .= (b "/\" (Bottom H)) "\/" (a "/\" (b "/\" ('not' b))) by A1, A3, Th82
  .= (b "/\" (Bottom H)) "\/" (a "/\" (Bottom H)) by A1, A4, Th82
  .= ((Bottom H) "/\" b) "\/" (a "/\" (Bottom H)) by A1, LATTICE3:15
  .= ((Bottom H) "/\" b) "\/" ((Bottom H) "/\" a) by A1, LATTICE3:15
  .= (Bottom H) "\/" ((Bottom H) "/\" a) by A1, Th3
  .= (Bottom H) "\/" (Bottom H) by A1, Th3
  .= Bottom H by A1, A2, YELLOW_0:24 ;
  hence 'not' (a "/\" b) >= ('not' a) "\/" ('not' b) by A1, Th82; :: thesis:
```

Learning From Mizar Proofs: YELLOW_5 36

```
let L be non empty Boolean RelStr ; :: thesis:
let a, b be Element of L; :: thesis:
A1: (a "\/" b) "/\" (('not' a) "/\" ('not' b)) = ((a "\/" b) "/\" ('not' a)) "/\"
('not' b) by LATTICE3:16
.= ((('not' a) "/\" a) "\/" (('not' a) "/\" b)) "/\" ('not' b) by WAYBEL_1:def 3
.= ((Bottom L) "\/" (('not' a) "/\" b)) "/\" ('not' b) by Th34
.= (('not' a) "/\" b) "/\" ('not' b) by WAYBEL_1:3
.= ('not' a) "/\" (b "/\" ('not' b)) by LATTICE3:16
.= ('not' a) "/\" (Bottom L) by Th34
.= Bottom L by WAYBEL_1:3 ;
(a "\/" b) "\/" (('not' a) "/\" ('not' b)) = a "\/" (b "\/" (('not' a) "/\"
('not' b))) by LATTICE3:14
.= a "\/" ((b "\/" ('not' a)) "/\" (b "\/" ('not' b))) by Th17
.= a "\/" ((b "\/" ('not' a)) "/\" (Top L)) by Th34
.= a "\/" (b "\/" ('not' a)) by WAYBEL_1:4
.= (a "\/" ('not' a)) "\/" b by LATTICE3:14
.= (Top L) "\/" b by Th34
.= Top L by WAYBEL_1:4 ;
then ('not' a) "/\" ('not' b) is_a_complement_of a "\/" b by A1, WAYBEL_1:def 23;
hence 'not' (a "\/" b) = ('not' a) "/\" ('not' b) by Th11; :: thesis:
A2: (a "/\" b) "/\" (('not' a) "\/" ('not' b)) = a "/\" (b "/\" (('not' a) "\/"
('not' b))) by LATTICE3:16
.= a "/\" ((b "/\" ('not' a)) "\/" (b "/\" ('not' b))) by WAYBEL_1:def 3
.= a "/\" ((b "/\" ('not' a)) "\/" (Bottom L)) by Th34
.= a "/\" (b "/\" ('not' a)) by WAYBEL_1:3
.= (a "/\" ('not' a)) "/\" b by LATTICE3:16
.= (Bottom L) "/\" b by Th34
.= Bottom L by WAYBEL_1:3 ;
(a "/\" b) "\/" (('not' a) "\/" ('not' b)) = ((a "/\" b) "\/" ('not' a)) "\/"
('not' b) by LATTICE3:14
.= ((('not' a) "\/" a) "/\" (('not' a) "\/" b)) "\/" ('not' b) by Th17
.= ((Top L) "/\" (('not' a) "\/" b)) "\/" ('not' b) by Th34
.= (('not' a) "\/" b) "\/" ('not' b) by WAYBEL_1:4
.= ('not' a) "\/" (b "\/" ('not' b)) by LATTICE3:14
.= ('not' a) "\/" (Top L) by Th34
.= Top L by WAYBEL_1:4 ;
then ('not' a) "\/" ('not' b) is_a_complement_of a "/\" b by A2, WAYBEL_1:def 23;
hence 'not' (a "/\" b) = ('not' a) "\/" ('not' b) by Th11; :: thesis:
```

Learning From Mizar Proofs

We work with

- Mizar Mathematical Library
- E prover
  - can be a hammer for theorem proving (ITP)
  - uses first order formula (fof) form of Mizar:

```
% Mizar problem: t85_waybel_1,waybel_1,3140,5
fof(t85_waybel_1, conjecture,  (! [A] :  ( ( ~ (v2_struct_0(A))  &  (v1_yellow_0(A) & l1_orders_2
(A)) )  =>  (v9_waybel_1(A) =>  (! [B] :  (m1_subset_1(B, u1_struct_0(A)) =>  (! [C] :  (m1_subse
t_1(C, u1_struct_0(A)) => r1_orders_2(A, k10_lattice3(A, k7_waybel_1(A, B), k7_waybel_1(A, C)), k
7_waybel_1(A, k11_lattice3(A, B, C)))) ) ) ) ) ) ) ).
fof(antisymmetry_r2_hidden, axiom,  (! [A, B] :  (r2_hidden(A, B) =>  ~ (r2_hidden(B, A)) ) ) ).
fof(cc10_struct_0, axiom,  (! [A] :  (l1_struct_0(A) =>  ( ( ~ (v2_struct_0(A))  & v7_struct_0(A)
)  => v13_struct_0(A, 1)) ) ) ).
```

Learning From Mizar Proofs

We work with

- Mizar Mathematical Library
- E prover
  - can be a hammer for theorem proving (ITP)
  - uses first order formula (FOF) form of Mizar
  - learns from E proof clauses in conjunctive normal form:

```
cnf(c_0_63,plain,(v2_struct_0(X1)|v1_lattice3(X1)|~l1_orders_2(X1)|~v9_waybel_1(X1))).# trainpos
cnf(c_0_52,plain,(v2_struct_0(X1)|v5_orders_2(X1)|~l1_orders_2(X1)|~v9_waybel_1(X1))).# trainpos
cnf(c_0_136,plain,(v2_struct_0(X1)|r1_orders_2(X1,X3,k7_waybel_1(X1,X2))|k11_lattice3(X1,X2,X3)
        !=k3_yellow_0(X1)|~v1_yellow_0(X1)|~l1_orders_2(X1)|~v9_waybel_1(X1)
        |~m1_subset_1(X3,u1_struct_0(X1))|~m1_subset_1(X2,u1_struct_0(X1)))).# trainpos
cnf(c_0_139,negated_conjecture,(m1_subset_1(k11_lattice3(esk1_0,esk2_0,esk3_0),u1_struct_0(esk1_0)))).# trainpos
```

ProofWatch: Results Overview

To prove YELLOW_5 Theorem 36, E considers

- 6550 given clauses without learning from proofs
- 5218 given clauses with learning from proofs (such as WAYBEL_1 85)

ProofWatch: Results Overview

On our benchmarks of 57987 Mizar theorems, E proves

- 21670 (37.3%) conjectures without learning from proofs
- 23192 (39.9%) conjectures with learning from proofs

This is a 7% improvement (and there are obviously improvements to try).

ProofWatch: Results Overview

- ProofWatch provides a proof-state vector that characterizes the conjecture's mathematical context.

ProofWatch: Watchlist History

- Clauses from related (E) proofs are put on the **Watchlist**.
  - E feature implemented by Stephan Schulz.
  - E checks if each clause it considers matches a watchlist clause.

```
cnf(c_0_63,plain,(v2_struct_0(X1)|v1_lattice3(X1)|~l1_orders_2(X1)|~v9_waybel_1(X1))).# trainpos
cnf(c_0_52,plain,(v2_struct_0(X1)|v5_orders_2(X1)|~l1_orders_2(X1)|~v9_waybel_1(X1))).# trainpos
cnf(c_0_136,plain,(v2_struct_0(X1)|r1_orders_2(X1,X3,k7_waybel_1(X1,X2))|k11_lattice3(X1,X2,X3)
        !=k3_yellow_0(X1)|~v1_yellow_0(X1)|~l1_orders_2(X1)|~v9_waybel_1(X1)
        |~m1_subset_1(X3,u1_struct_0(X1))|~m1_subset_1(X2,u1_struct_0(X1)))).# trainpos
cnf(c_0_139,negated_conjecture,(m1_subset_1(k11_lattice3(esk1_0,esk2_0,esk3_0),u1_struct_0(esk1_0)))).# trainpos
```

ProofWatch: Watchlist History

- E proof clauses are put on the **Watchlist**.
  - E feature implemented by Stephan Schulz.
  - E checks if each clause it considers matches a watchlist clause.
- Hint lists (Veroff 96)
  - provide a similar feature in Prover9 and Otter
  - used for proving extensions of AIM (Abelian Inner Mapping) conjecture in loop theory by Bob Veroff
  - have obtained very long proofs (1000+ steps)

ProofWatch: Watchlist Usage

What goes on the watchlist?

- Related or similar proofs' clauses
- Proof sketches
- Conjectured lemmas/sub-goals
- Generally useful clauses or "mathematical tricks"

ProofWatch: Watchlist Usage

1. Matching clauses (subsumption) is **symbolic** AI.
2. Matching with proof completion is **statistical** AI.
3. The vectorial *proofstate* characterization can work with standard ML methods.
4. Watchlist provides logical interface between AI, human, or ITP context and the ATP:

ProofWatch: Experiments

We do the first benchmark of E's Watchlist. Specifically:

- how to include the watchlist in E strategies?
- how to build the watchlist?
- *matching* **watchlist clauses** modulo skolem names (e.g., $esk(A, B) \sqsubseteq eskfoobar(A, B)$)
- dynamic weighting of **hints** by proof completion (aka 'relevance')
- k-NN recommendation of prior proofs based on conjecture features

E Basics: Goal

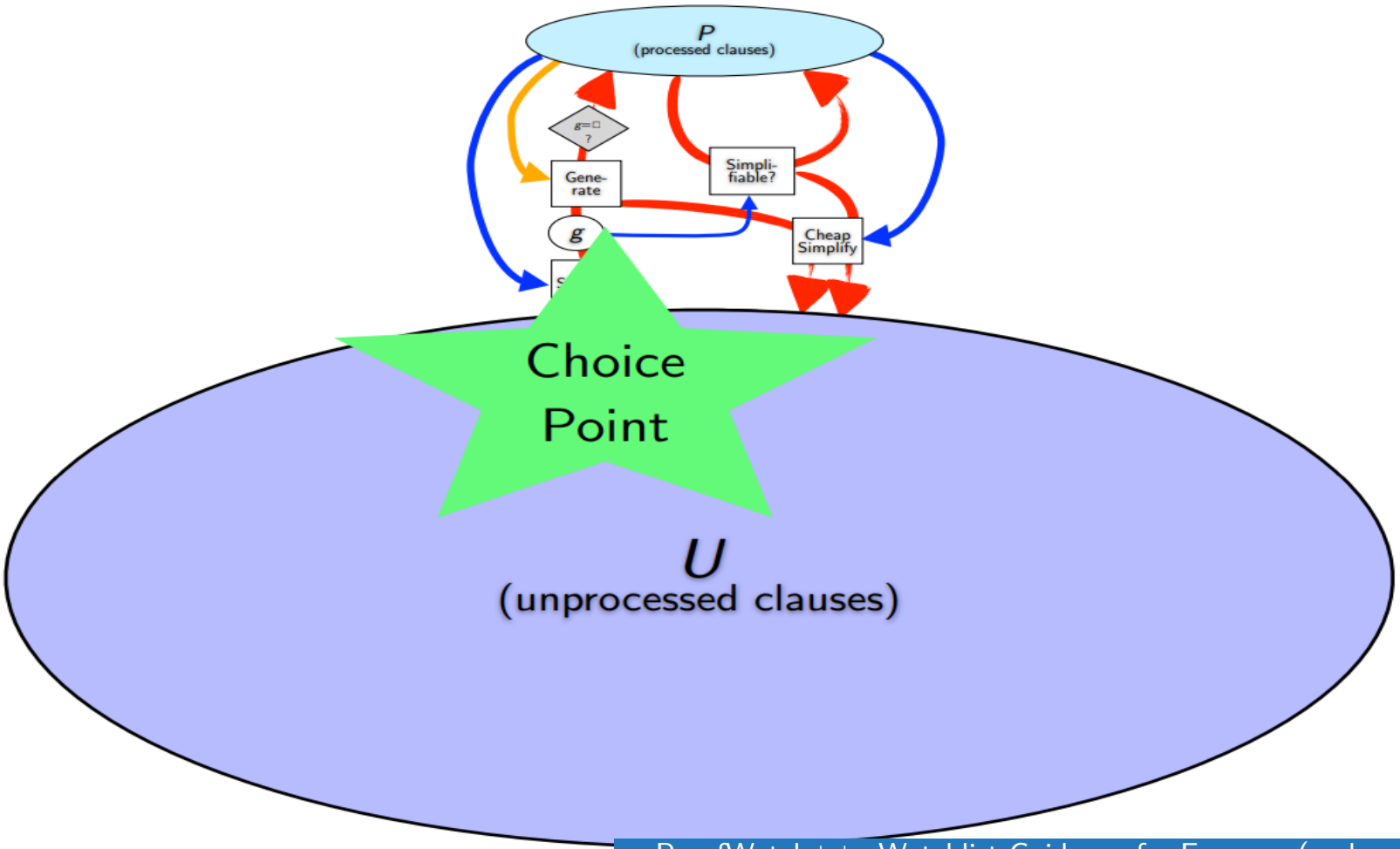
- E's a saturation-based refutational theorem prover (using superposition calculus)
- To prove P, E attempts to prove $\{\neg P \cup \text{Premises}\} \rightarrow \bot$

E Basics: Given Clause Loop

- E has two sets of clauses:
  - Processed clauses
  - Unprocessed clauses
- E's modus operandi is the **given clause loop**:

```
while (no proof found)
{
  select a given clauses
  apply inference rules to selected clauses
  process inferred clauses
}
```

Motivation


P
(processed clauses)
g=□ ?
Gene-rate
Simpli-fiable?
g
Cheap Simplify
Choice Point
U
(unprocessed clauses)

Basic Watchlist Mechanism: Given Clause Selection

- **watchlist**: list of clauses, hints.
- Check if each clause subsumes a **hint** (clause $\sqsubseteq$ hint)
- If yes, boost clause priority (and thus score), making selection more likely.

Basic Watchlist Mechanism: Clause Evaluation Functions

- E chooses given clauses by weighted round-robbin selection of clause evaluation functions (CEFs),
- Clause evaluation functions
  - partition clauses into queues based on *priority*
  - order queues based on clause weight

Basic Watchlist Mechanism: Clause Evaluation Functions

An example E strategy:

```
-tKBO -H(2*Clauseweight(PreferWatchlist,20,9999,4)
        ,4*FIFOWeight(PreferUnit))
```

Where

- **Priority** functions: PreferWatchlist, PreferUnit
- **Weight** functions: Clauseweight, FIFOWeight

Baseline 02: An Actual Evolved Strategy File

```
--definitional-cnf=24 --split-aggressive --simul-par
amod --forward-context-sr --destructive-er-aggressive
--destructive-er --prefer-initial-clauses -tKBO -winv
freqrank -c1 -Ginvfreq -F1 --delete-bad-limit=150000000
-WSelectMaxLComplexAvoidPosPred
-H(1*ConjectureTermPrefixWeight(DeferSOS,1,3,0.1,5,0,0.1,1
,1*ConjectureTermPrefixWeight(DeferSOS,1,3,0.5,100,0,0.2,0
,1*Refinedweight(PreferWatchlist,4,300,4,4,0.7)
,1*RelevanceLevelWeight2(PreferProcessed,0,1,2,1,1,1,200,20
,1*StaggeredWeight(DeferSOS,1)
,1*SymbolTypeweight(DeferSOS,18,7,-2,5,9999.9,2,1.5)
,2*Clauseweight(PreferWatchlist,20,9999,4)
,2*ConjectureSymbolWeight(DeferSOS,9999,20,50,-1,50,3,3,0.5
,2*StaggeredWeight(Defer
```

Dynamic Watchlist Mechanism

- Multiple watchlists: one for each loaded proof.
- Keep track of which proofs **hints** come from.
- Count what % of each proof has been covered (subsumed)
- Assign more priority for higher completion %.
  - –> closer to contradiction
  - –> closer to current proof
- Can represent proof state.
- The **PreferWatchlistRelevant** priority function.

Dynamic Watchlist Mechanism: Relevance Inheriting

Probably a good idea to keep following a good hint:

- *Inherit watchlist priority from parents*

Priority queues are absolute, so

- Decrease inheritance each generation with multiplicative decay.
- Drop to zero below a threshold

Dynamic Watchlist Mechanism: Relevance Calculation

For a clause C matching at least one watchlist W_i:

$$relevance_0(C) = \max_{W \in \{W_i : C \sqsubseteq W_i\}} \left(\frac{progress(W)}{|W|} \right)$$

Where $progress(W)$ is how many clauses in watchlist W have been subsumed.

Dynamic Watchlist Mechanism: Inheritance Calculation

With decay, for $\delta < 1$,

$$relevance_1(C) = relevance_0(C) + \delta * \operatorname*{avg}_{D \in parents(C)} \Big(relevance_1(D) \Big)$$

Aditionally, reset to 0 if $relevance_1(C) < \alpha$ and $\frac{relevance_1(C)}{length(C)} < \beta$ for *threshold* parameters $\alpha, \beta > 0$.

Baseline Strategies

- We use 57897 Mizar40 conjectures with premises pre-selected.
- We previously evolved 32 strategies with a coverage of 24702 proofs (in 5s each).
- We use a *top 5 greedy cover* of strategies:
  Strategies A-E.
  In 10s they together solve 21670 problems.
  In 5 s they together solve 21122 problems.

Using the Watchlist

E native option:

- Stephan Schulz's -**xUseWatchlistEvo** (*evo*)

Using the Watchlist

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- Stephan Schulz's **-xUseWatchlistEvo** (*evo*)

Some options (modifying baseline strategies):

- All **priority functions** -> **PreferWatchlist** (*pref*)
- Always PreferWatchlist, default to other priority function if not on watchilst (*uwl*).
- Match Skolems by name and arity only (on top of *pref*) (*ska*).
- Priority functions -> **PreferWatchlistRelevant** (*dyn*).
- Add relevance inheritance to *dyn* (*dyndec*).

Using the Watchlist

E native option:

- Stephan Schulz's -**xUseWatchlistEvo** (*evo*)

Some options (modifying baseline strategies):

- All **priority functions** -> **PreferWatchlist** (*pref*)
- Always PreferWatchlist, default to other priority function if not on watchilst (*uwl*).
- Match Skolems by name and arity only (on top of *pref*) (*ska*).
- Priority functions -> **PreferWatchlistRelevant** (*dyn*).
- Add relevance inheritance to *dyn* (*dyndec*).

And for comparison:

- Priority functions -> **ConstPrio** (*const*)

Creating the Watchlist

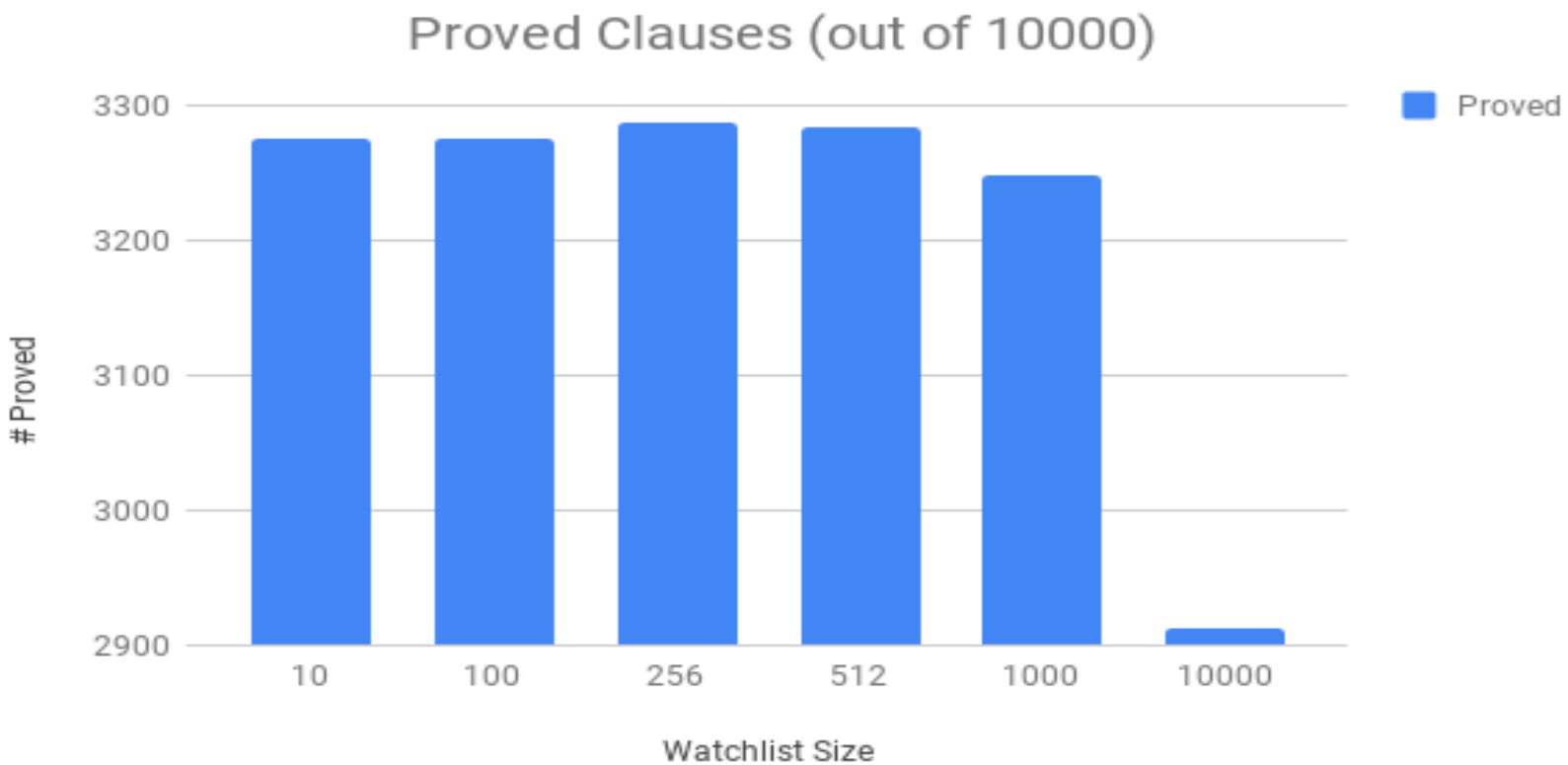
- Use most frequent clause
- Use proof clauses from the conjecture's Mizar article (size 1-4000)
- Use k-NN to suggest clauses
- Use k-NN to suggest proofs

Informal/Preliminary Testing

- Clauses from proofs by same strategy seem more useful.
- Not removing matched **hints** helps with small watchlists.
- Defaultweight(PreferWatchlist) sucks: **weight** functions are good.

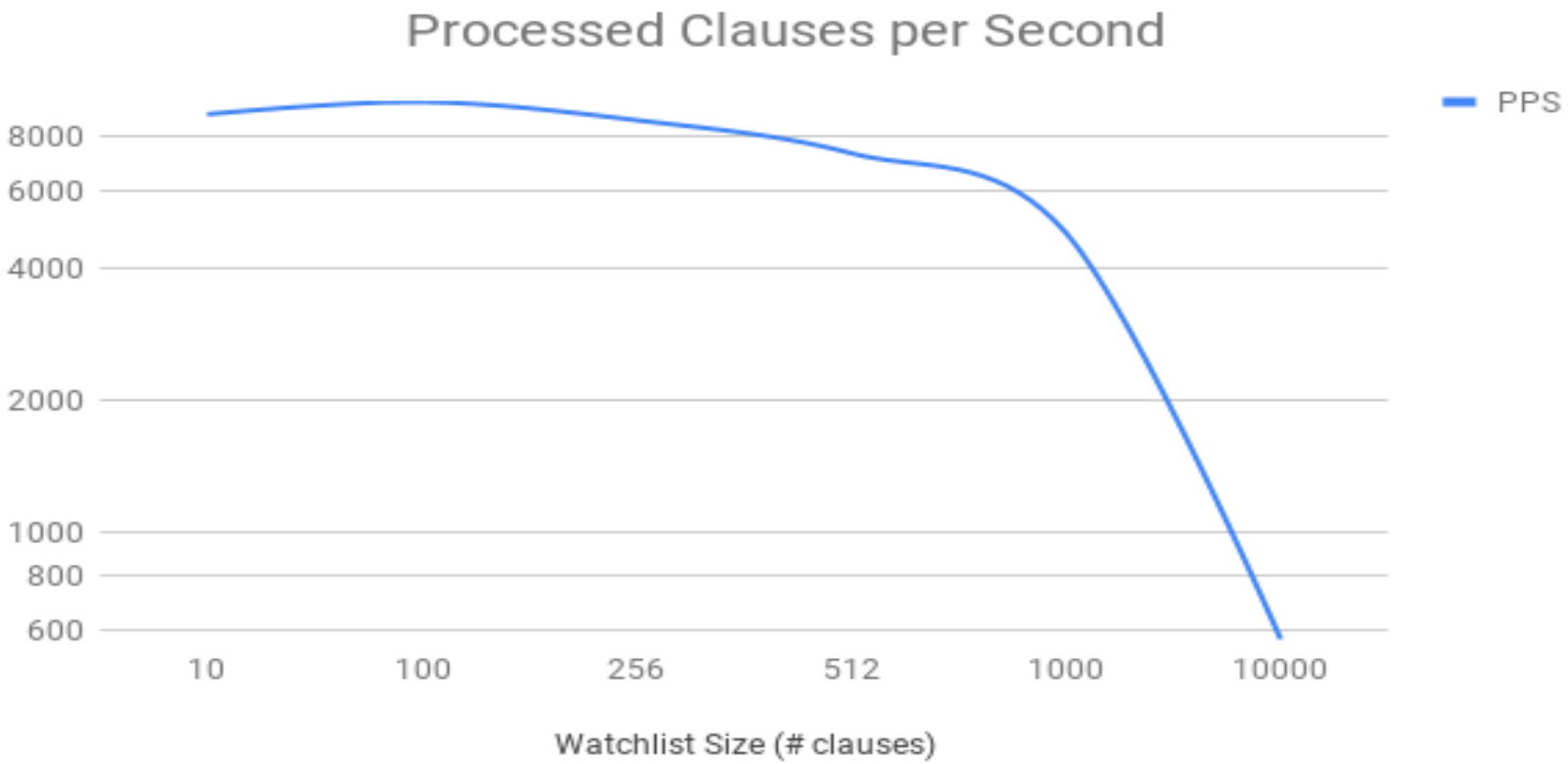
Small Most Frequent Clauses Test

Results of *prefA* run with **hints** taken from all articles.



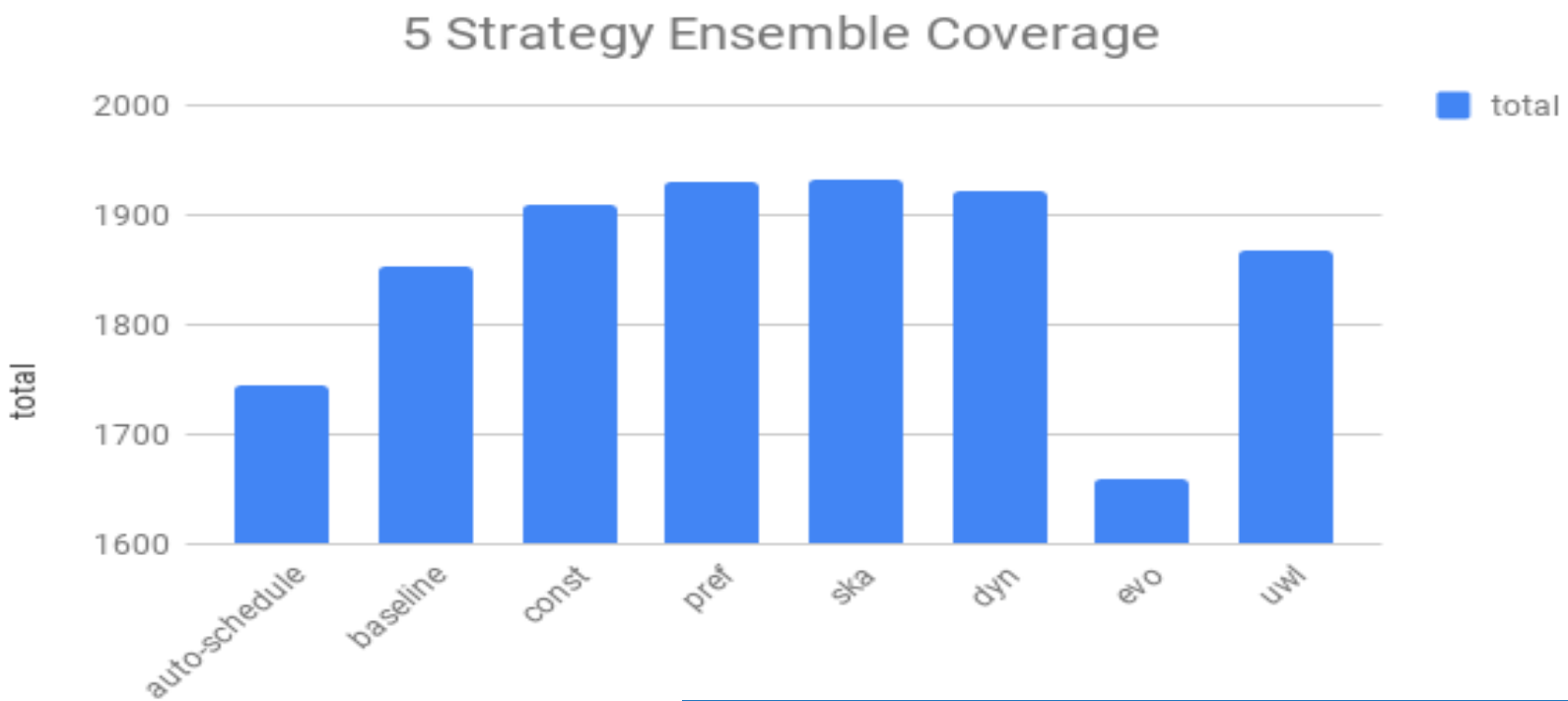
Small Most Frequent Clauses Test

Results of *prefA* run with **hints** taken from all articles.

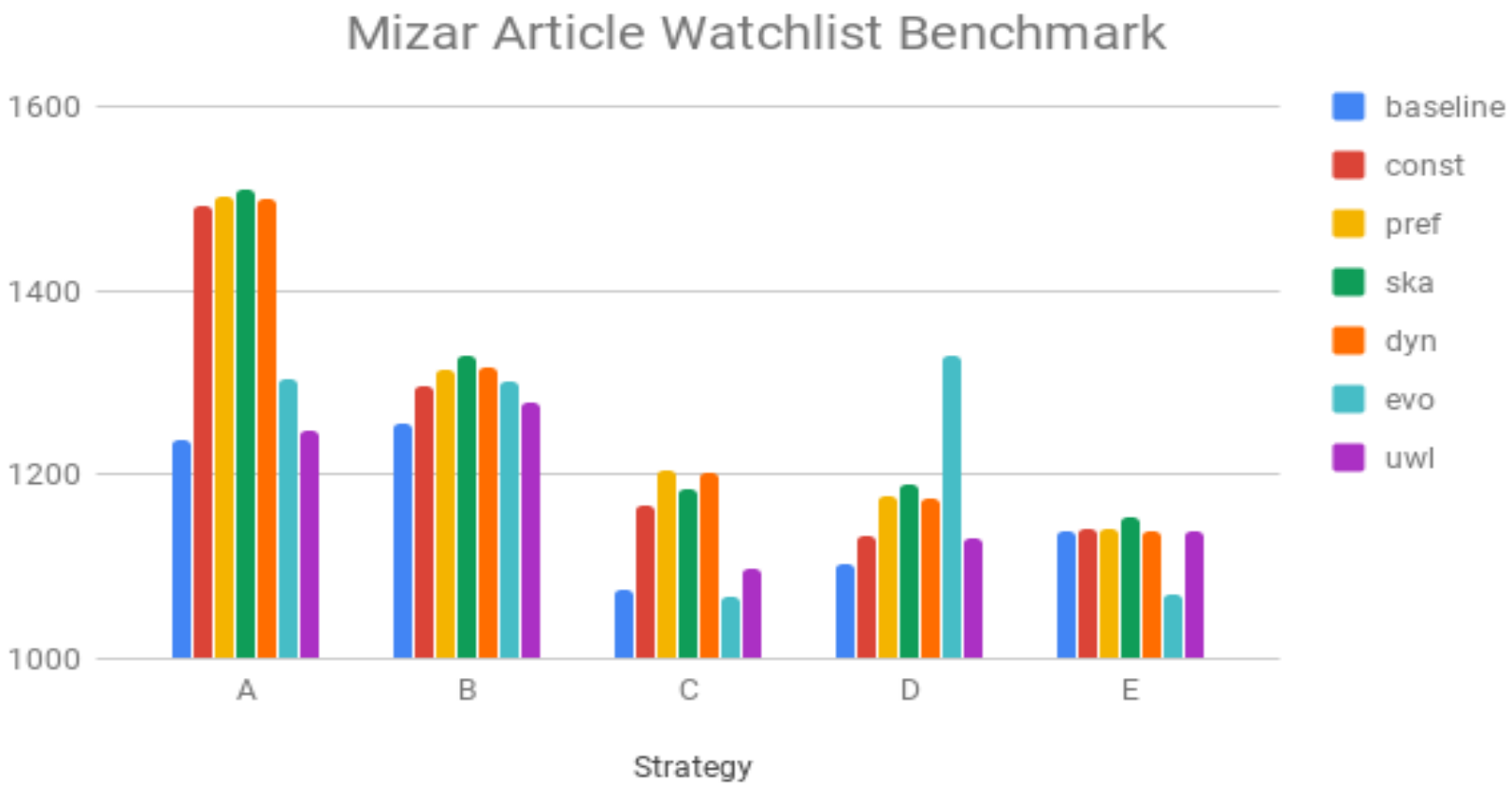


Mizar-article Watchlist Benchmark

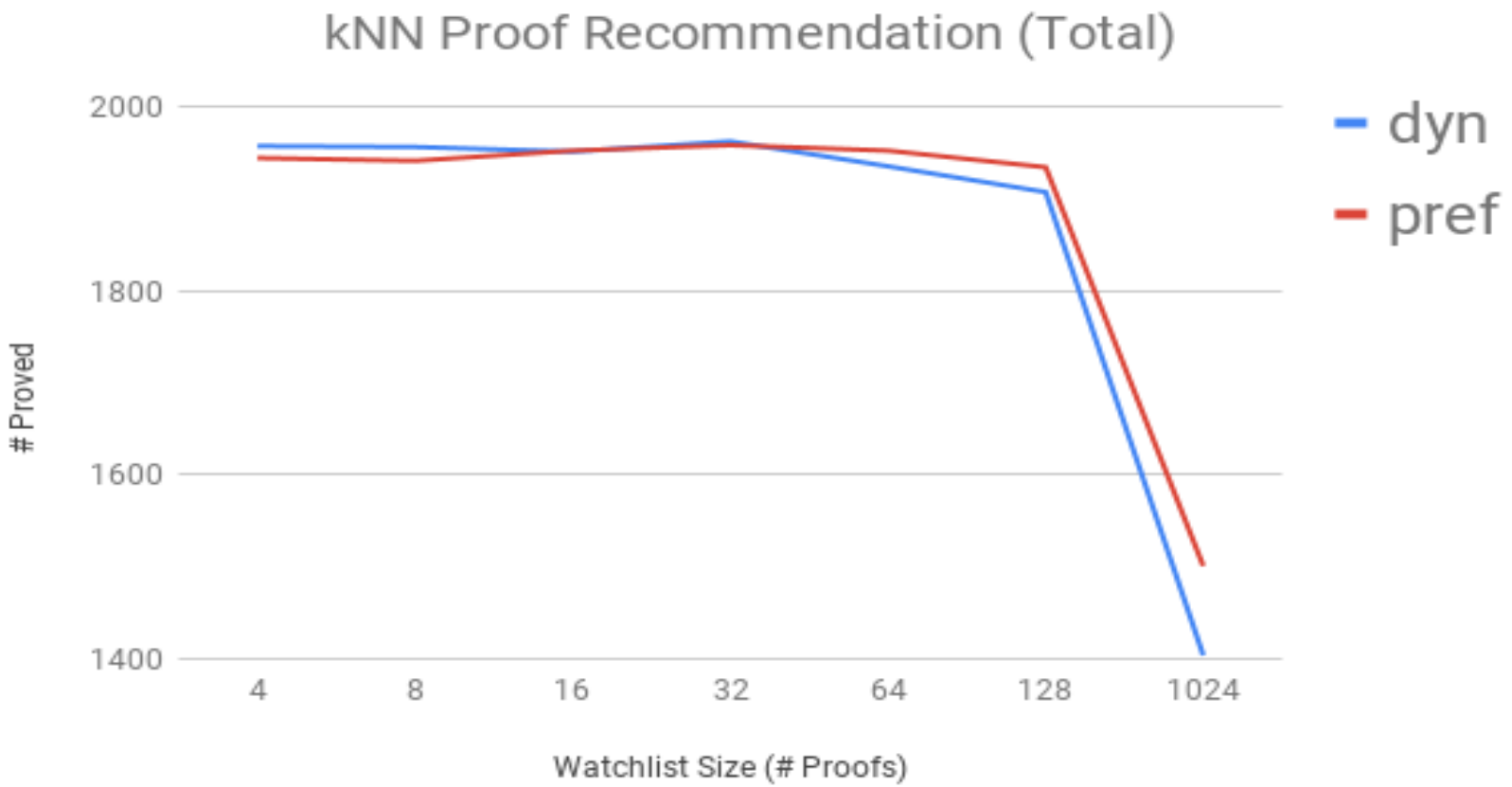
The strategies are run for 10s each on a random sample of 5000 conjectures.



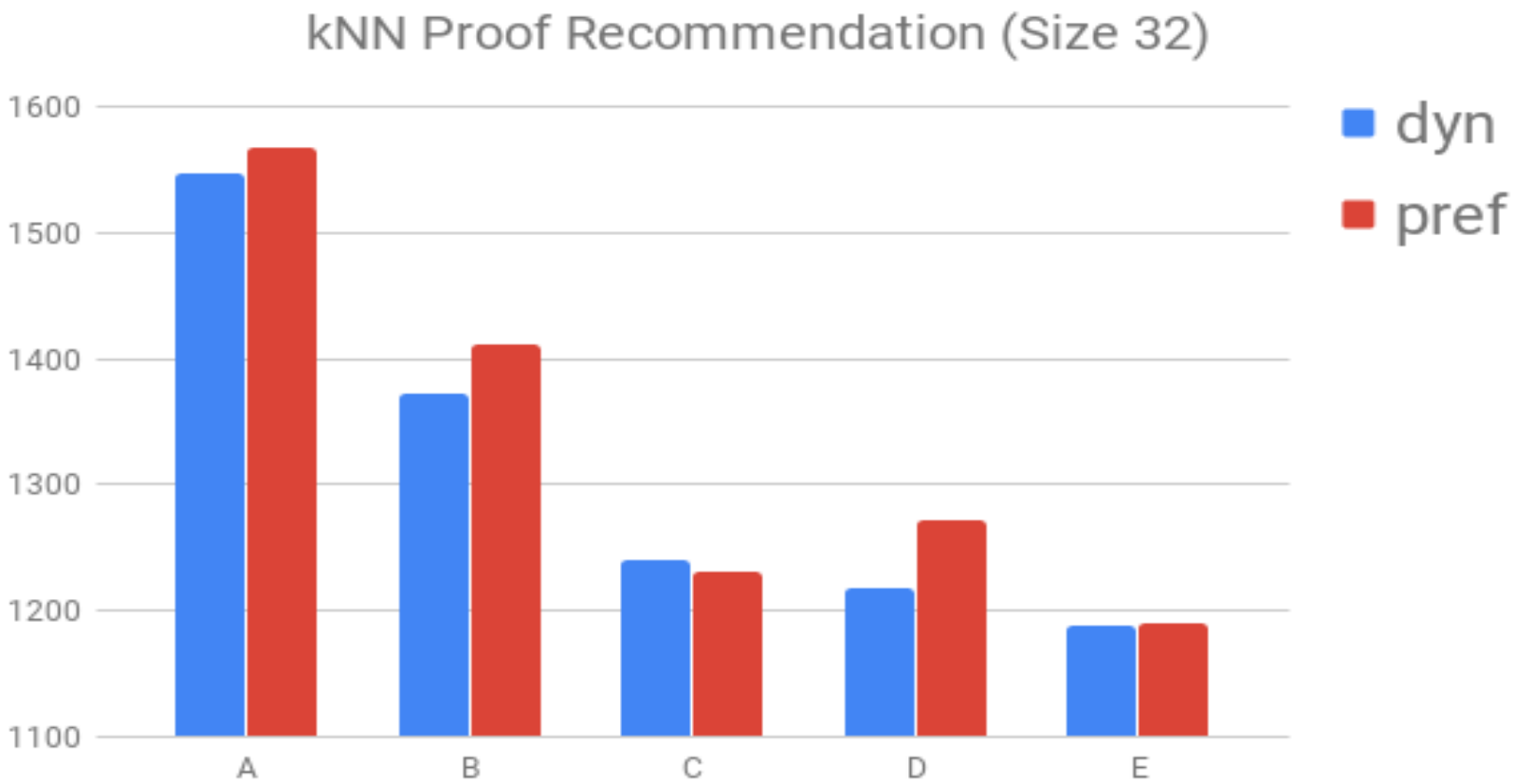
Mizar-article Watchlist Benchmark



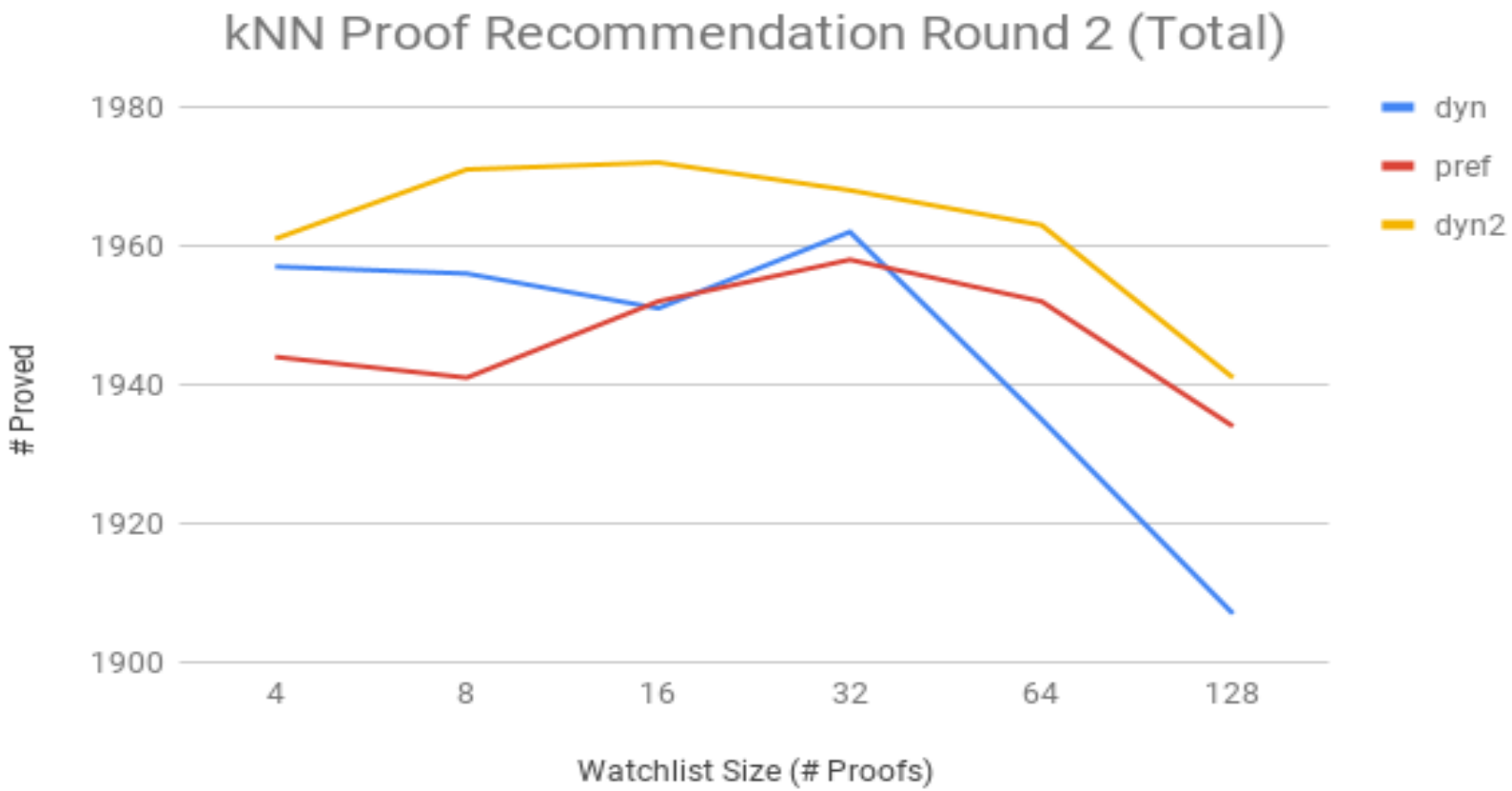
k-NN Dynamic Watchlist Round 1



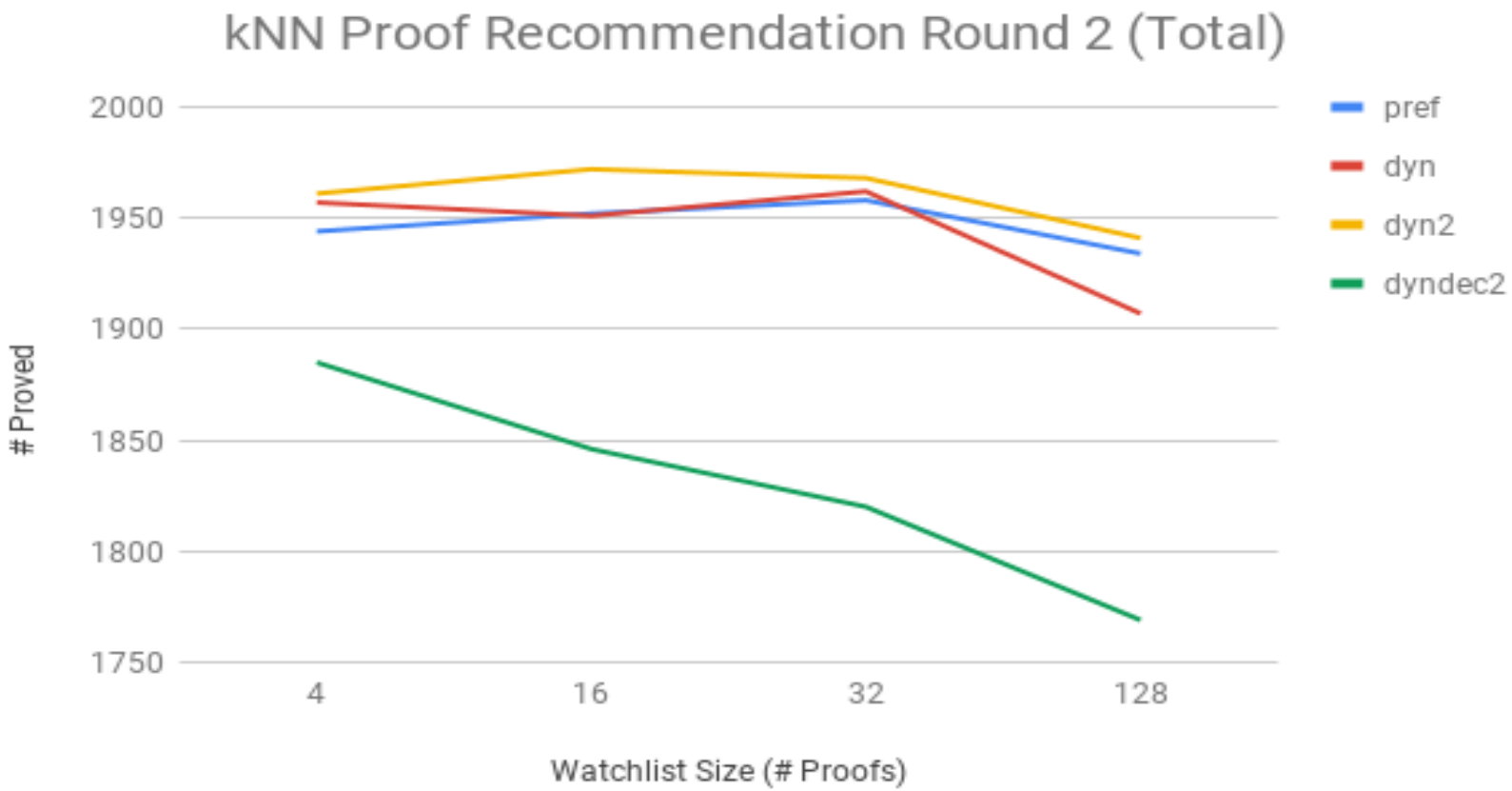
k-NN Dynamic Watchlist Round 1



k-NN Dynamic Watchlist Round 2

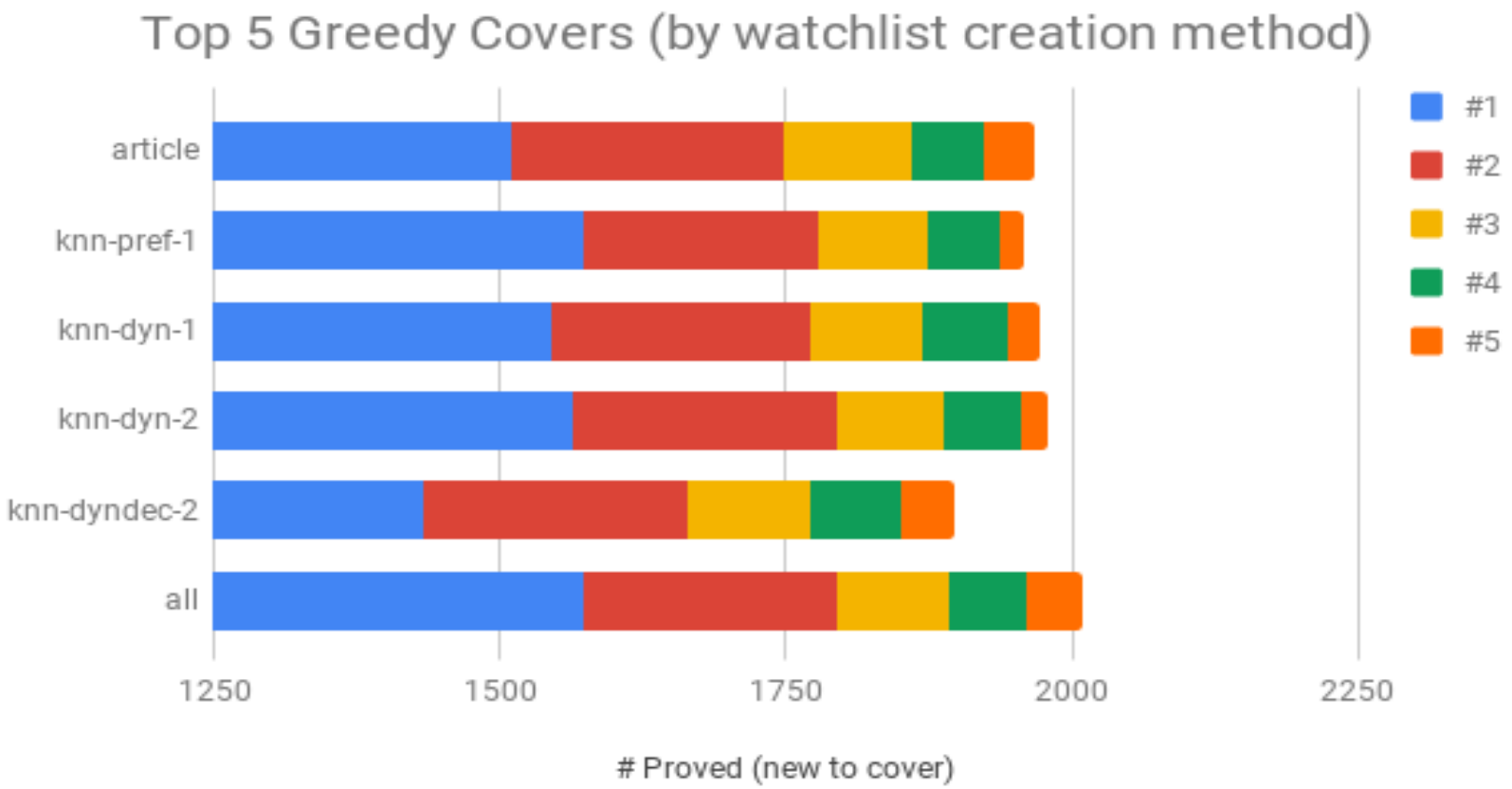


k-NN Dynamic Inheritance Watchlist



Divergence from baseline

Grand Total Results



Grand Total Results

total	knn-prefA_16	dyn2C_16	dyndecD_16	dynE_4	dyndecA_128
2007	1565	230	97	68	47
23192	18583	2553	1050	584	422
23192	18583	14486	14514	13532	15916

Table: Top: Cumulative sum of the 5000 test set greedy cover. Recall the baseline covered 1853 and *const* 1910. Bottom: Greedy cover run on the full dataset, cumulative and total proved. The top 5 baseline strategies covered 21657. Thus there's an improvement of 7%.

De Morgan's laws for Boolean lattices

```
theorem Th36: :: YELLOW_5:36
  for L being non empty Boolean RelStr
  for a, b being Element of L holds
  ( 'not' (a "\/" b) = ('not' a) "/\" ('not' b) & 'not' (a "/\" b) = ('not' a) "\/" ('not' b) )
```

helped significantly by

```
theorem :: WAYBEL_1:85
  for H being non empty lower-bounded RelStr st H is Heyting holds
  for a, b being Element of H holds 'not' (a "/\" b) >= ('not' a) "\/" ('not' b)
```

Note the weaker assumptions on H: just lower bounded and Heyting. Yet, 62 (80.5%) of the 77 clauses from the proof of `WAYBEL_1:85` are matched.

De Morgan's laws for Boolean lattices - Statistics

- 32 related proofs results in 2220 clauses on the watchlists.
- The dynamically guided proof search takes 5218 (nontrivial) given clause loops
- The ATP proof is 436 inferences long
- 194 given clauses match the watchlist in the proof search
- 120 (61.8%) of them end up being part of the proof
- 27.5% of the proof steps guided by the watchlist
- The proof search without the watchlist takes 6550 nontrivial given clause loops (25.5% more)

Final proof progress

0	0.438	42/96	1	0.727	56/77	2	0.865	45/52	3	0.360	9/25
4	0.750	51/68	5	0.259	7/27	6	**0.805**	**62/77**	7	0.302	73/242
8	0.652	15/23	9	0.286	8/28	10	0.259	7/27	11	0.338	24/71
12	0.680	17/25	13	0.509	27/53	14	0.357	10/28	15	0.568	25/44
16	0.703	52/74	17	0.029	8/272	18	0.379	33/87	19	0.424	14/33
20	0.471	16/34	21	0.323	20/62	22	0.333	7/21	23	0.520	26/50
24	0.524	22/42	25	0.523	45/86	26	0.462	6/13	27	0.370	20/54
28	0.411	30/73	29	0.364	20/55	30	0.571	16/28	31	0.357	10/28

Table: Final state of the proof progress for the (serially numbered) 32 proofs loaded to guide the proof of `YELLOW_5:36`. We show the computed ratio and the number of matched and all clauses.

ProofWatch Only Example

```
theorem :: BOOLEALG:68
  for L being B_Lattice
  for X, Y being Element of L holds (X \+\ Y) ` = (X "/\" Y) "\/" ((X `) "/\" (Y `))
proof end;
```

using 272/285, 283/350, and 155/233 clauses from:

```
theorem Th33: :: BOOLEALG:33
  for L being B_Lattice
  for X, Y being Element of L holds X \ (X "/\" Y) = X \ Y
proof end;

theorem :: BOOLEALG:67
  for L being B_Lattice
  for X, Y, Z being Element of L holds (X \+\ Y) \+\ Z = X \+\ (Y \+\ Z)
proof end;

theorem Th23: :: LATTICES:23
  for L being B_Lattice
  for a, b being Element of L holds (a "/\" b) ` = (a `) "\/" (b `)
proof end;
```

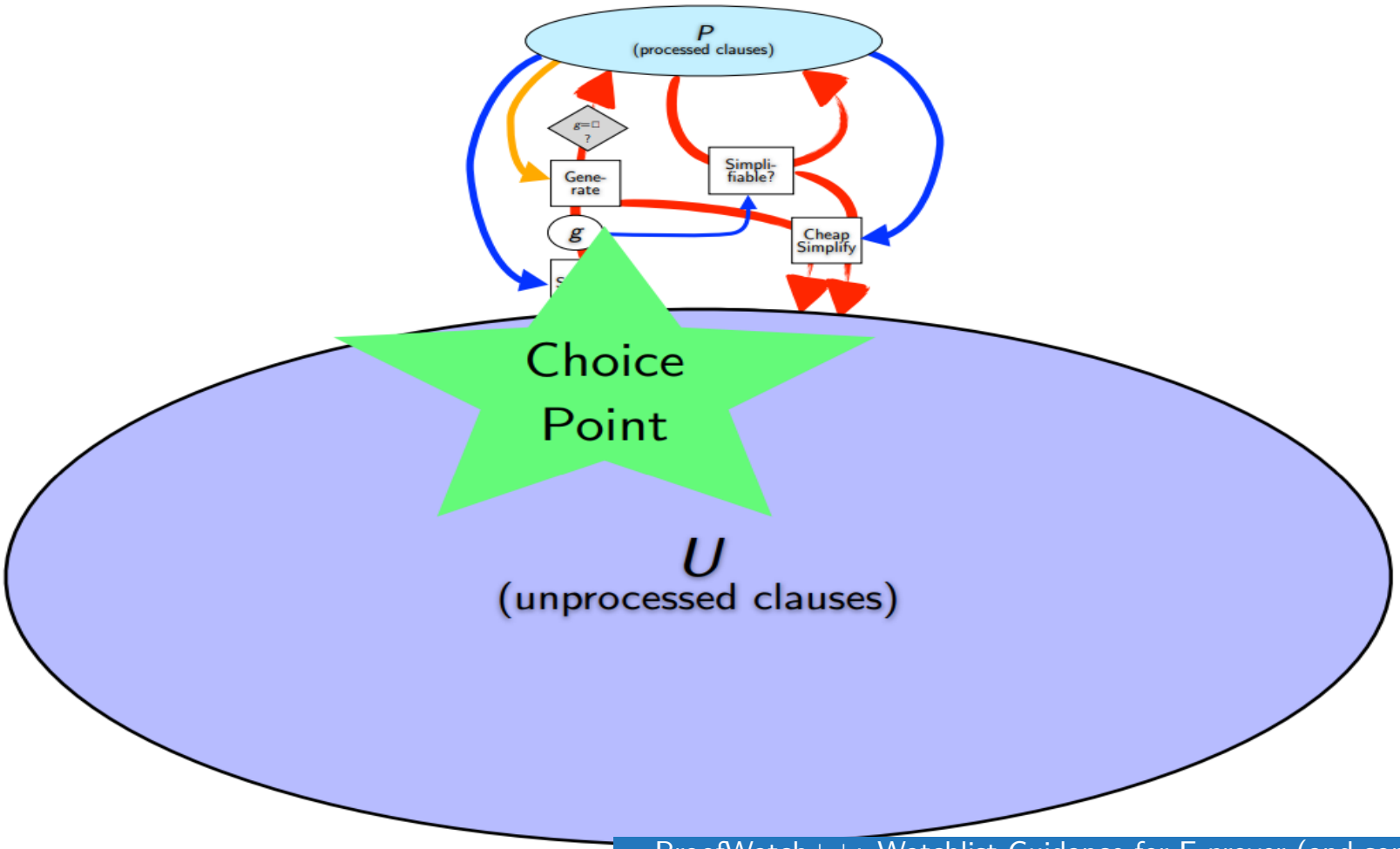
Proof Progress

```
#   watchlist    0: 0.714 (       5 /       7)
#   watchlist    1: 0.500 (      19 /      38)
#   watchlist    2: 0.649 (      72 /     111)
#   watchlist    3: 0.945 (      69 /      73)
#   watchlist    4: 0.618 (      21 /      34)
#   watchlist    5: 0.954 (     272 /     285)
#   watchlist    6: 0.565 (      13 /      23)
#   watchlist    7: 0.765 (      13 /      17)
#   watchlist    8: 0.484 (      30 /      62)
#   watchlist    9: 0.639 (      69 /     108)
#   watchlist   10: 0.809 (     283 /     350)
#   watchlist   11: 0.460 (      40 /      87)
#   watchlist   12: 0.667 (      52 /      78)
#   watchlist   13: 0.765 (      78 /     102)
#   watchlist   14: 0.833 (     110 /     132)
#   watchlist   15: 0.665 (     155 /     233)
```

Conclusion (of ProofWatch talk)

- Upgraded E's watchlist to include dynamic proof-completion and relevance inheritance.
  - and to output a proof-state vector with contextual information.
- Benchmarked E's watchlist feature on Mizar Mathematical Library.
- Demonstrated k-NN proof recommendation as effective for watchlist creation.
  - in successive rounds
- ProofWatch obtains 7% improvement over strong non-watchlist E strategies.

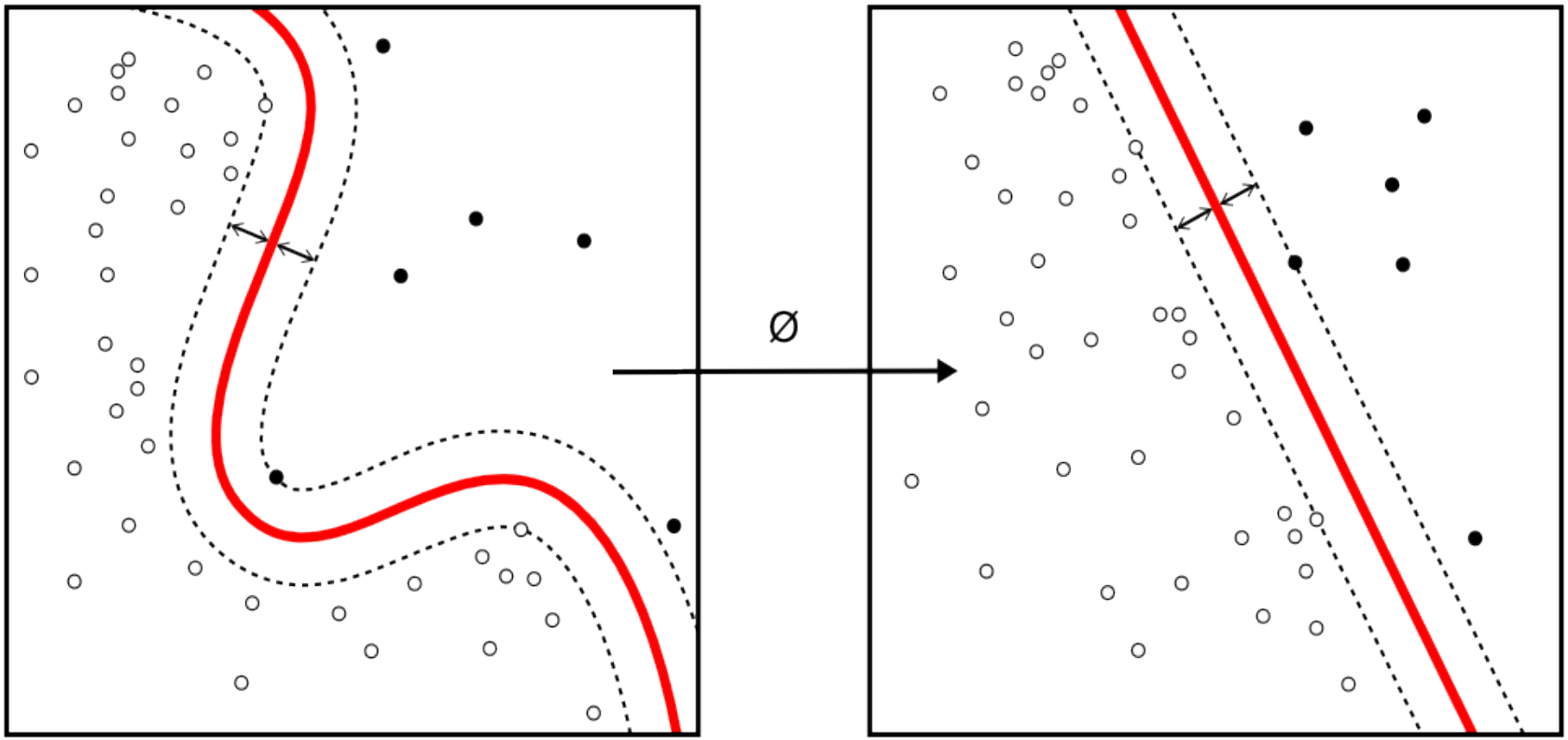
Motivation: Use Machine Learning!



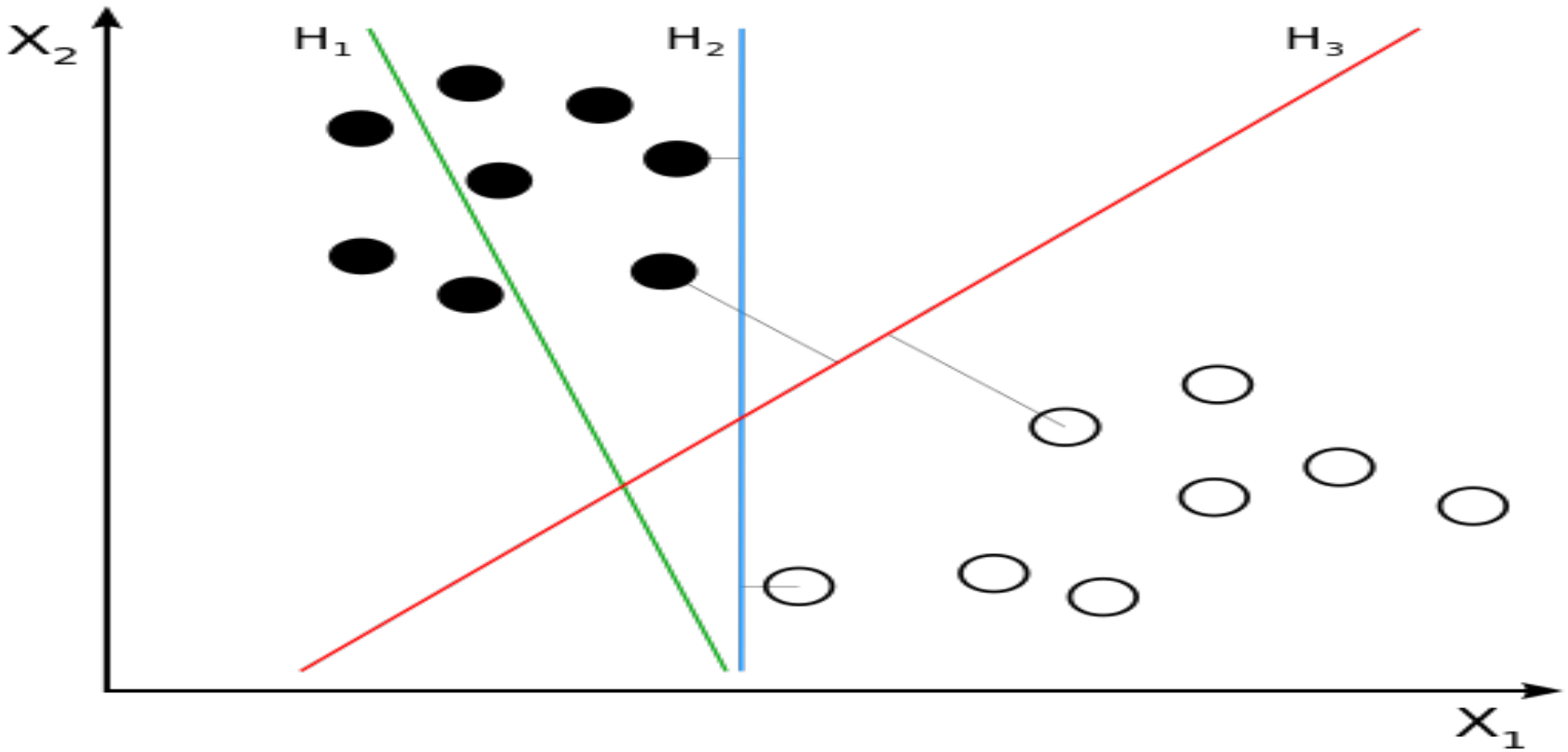
ENIGMA: Efficient learNing-based Inference Guiding MAchine

- Learn a linear model to *classify* clauses as **good** or **bad**.
- Train on proof-data:
  - C_1 : Feature_Vector # trainpos
  - C_2 : Feature_Vector # trainneg
  - . . .
- Turn ENIGMA prediction into weight function (as a CEF):
  - weight$(C, M) = \gamma$length(C) + predict-weight(C, M)
  - Predicted weighst are 1 (positive) or 10 (not positive)

ENIGMA: LIBLINEAR Support Vector Machine



ENIGMA: LIBLINEAR Support Vector Machine



Can we combine ProofWatch + ENIGMA?

- ProofWatch: logically-grounded given clause selection (with some ML)
- ENIGMA: Machine Learned model based given clause selection

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- ProofWatch: logically-grounded given clause selection (with some ML)
- ENIGMA: Machine Learned model based given clause selection
- Key: feed Proof Vectors to ENIGMA.
  - Represents current proof state wrt a set of proofs
  - Add this to ENIGMA features

Proof Progress

```
#   watchlist    0: 0.714 (       5 /       7)
#   watchlist    1: 0.500 (      19 /      38)
#   watchlist    2: 0.649 (      72 /     111)
#   watchlist    3: 0.945 (      69 /      73)
#   watchlist    4: 0.618 (      21 /      34)
#   watchlist    5: 0.954 (     272 /     285)
#   watchlist    6: 0.565 (      13 /      23)
#   watchlist    7: 0.765 (      13 /      17)
#   watchlist    8: 0.484 (      30 /      62)
#   watchlist    9: 0.639 (      69 /     108)
#   watchlist   10: 0.809 (     283 /     350)
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```
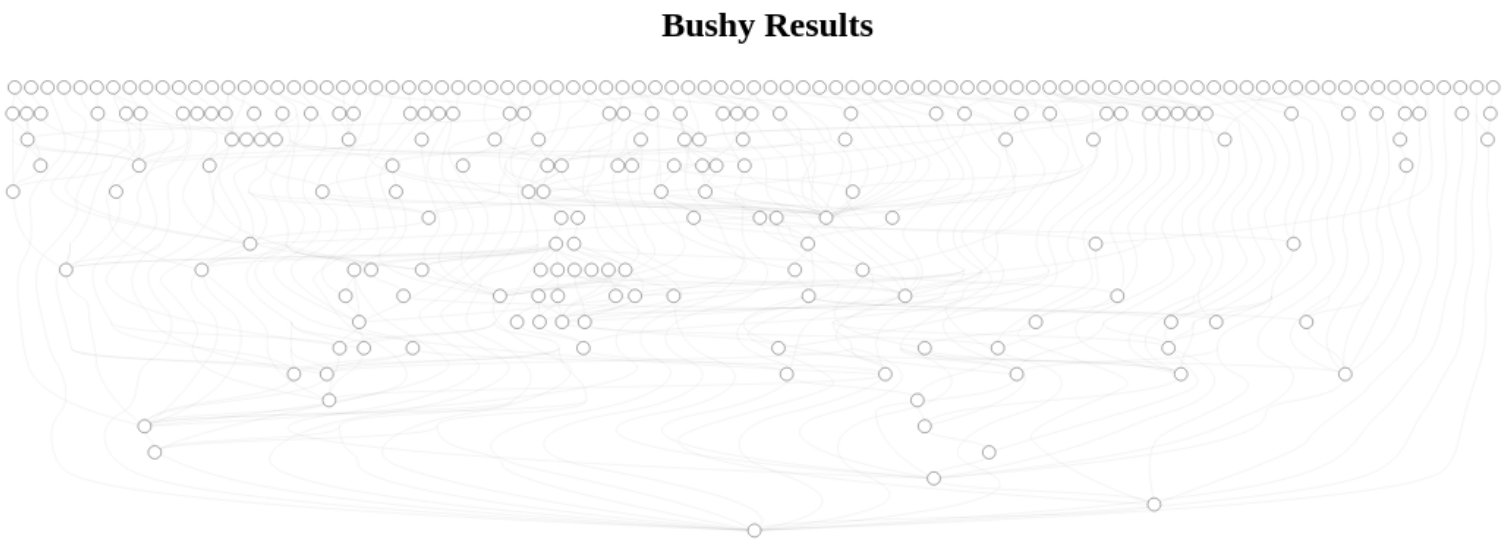

First Test: MPTP Challenge

Prove Bolzano-Weierstrass theorem.



MPTP Challenge Results

Run E for 15 seconds or 40,000 given clause loops. With 10 strategies. 252 problems total.

- 195 – Baseline strategies
- 200 – Watchlist strategies (*pref*)
- 202 – ENIGMA
- 205 – ENIGMA + Watchlist strategies

Next challenge: MPTP2078

- Also Bolzano-Weierstrass theorem.
- But all theorems from the 33 relevant Mizar articles.
- Baseline strategies prove 1461: too much for ProofWatch to handle

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- Also Bolzano-Weierstrass theorem.
- But all theorems from the 33 relevant Mizar articles.
- Baseline strategies prove 1461: too much for ProofWatch to handle
- How to construct good *proof vectors*?

Proof Vector Construction Ideas

- Use k-NN as in ProofWatch: large proof vector with many zeros in each case.
- Use K-means to cluster theorems/proofs into sufficiently small sets.
- Seek generic 'basis' vector for all 1461 proofs.
  - Initial k-mediods experiment failed :'(.